FALL 2019: MATH 558 QUIZ 3 SOLUTIONS

Each question is worth 5 points.

1. State (carefully) the division algorithm for set of integers.

Solution. Division Algorithm: Let a, b be integers, with $b > 0$. Then there exists unique integers q and r such that :

(i) $a = bq + r$
(ii) $0 \leq r < b$.

2. Use the Euclidean algorithm to find the greatest common divisor of 12 and 44. Then write this greatest common divisor as an integer combination of 12 and 44. You must show all steps to receive full credit.

Solution. Repeated applications of the division algorithm yields:

$$44 = 12 \cdot 3 + 8$$
$$12 = 8 \cdot 1 + 4$$
$$8 = 4 \cdot 2 + 0.$$

Thus, 4 is the last non-zero remainder and is therefore the GCD of 12 and 44. To write 4 as an integer combination of 12 and 44, we use the equations above in reverse order, starting with the next to last equation:

$$4 = 1 \cdot 12 + (-1) \cdot 8$$
$$4 = 1 \cdot 12 + (-1) \cdot (44 - 3 \cdot 12)$$
$$4 = 4 \cdot 12 + (-1) \cdot 44.$$